Midterm 1 - Math 217

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Name: ________________________________

Show work for all of your answers. Cross out any work that you do not want graded. **Be sure to read problem statements carefully.**

Problem	**Grade**	**Max**
1		15
2		20
3		15
4		20
5		15
6		15
Total		100

(1) Find a set of vectors in $\mathbb{R}^4$ which spans the solution set of the equation

$$2x_1 - x_2 + 4x_3 + 2x_4 = 0.$$

(2) (a) Find the inverse of the matrix

$$\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 2 & 1 \end{bmatrix}.$$

(b) Solve the following equation:

$$\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ -1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

(3) Let $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a map which rotates points in $\mathbb{R}^2$ counterclockwise around the origin by an angle of $3\pi/4$. Is f a linear transformation? If not, explain why not. If so, write down the standard matrix for f.

(4) Draw the solution set for the following linear system. Indicate the coordinates of two points in the solution set.

$$\begin{aligned} 3x_1 - x_2 &= 5 \\ 9x_1 - 3x_2 &= 15 \end{aligned}$$

(5) Let $T\colon \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation defined by

$$\begin{aligned} T(\mathbf{e}_1) &= \mathbf{e}_1 + \mathbf{e}_2 \\ T(\mathbf{e}_2) &= \mathbf{e}_2 + \mathbf{e}_3 \\ T(\mathbf{e}_3) &= \mathbf{e}_3 + \mathbf{e}_4 \\ T(\mathbf{e}_4) &= \mathbf{e}_4 + \mathbf{e}_1. \end{aligned}$$

Is T invertible? If not, explain why not. If so, write down a linear transformation that is an inverse to T.

(In this problem, $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ and $\mathbf{e}_4$ denote the column vectors of the 4×4 identity matrix.)

(6) Indicate whether the following assertions are true or false. If an assertion is true, explain why. If an assertion is false, give a counterexample.

- Let A and B be matrices such that the matrix product AB is defined. If the matrix transformations $\mathbf{x} \mapsto A\mathbf{x}$ and $\mathbf{x} \mapsto B\mathbf{x}$ are both one-to-one, then the matrix transformation $\mathbf{x} \mapsto AB\mathbf{x}$ must be one-to-one.

- If $\mathbf{b}$ is a vector in $\mathbb{R}^m$, and C is an $m \times n$ matrix that has a pivot position in every column, then the equation $C\mathbf{x} = \mathbf{b}$ must have a solution.

(6) (cont.)

- Let D be a 3×3 matrix. Then there must exist some sequence of real numbers $a_0, a_1, \ldots, a_n$, not all zero, such that
$$a_n D^n + a_{n-1} D^{n-1} + \ldots + a_1 D + a_0 I_3 = 0.$$