

Midterm 2 - Math 217

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Name: _____

Show work for all of your answers. Cross out any work that you do not want graded. **Be sure to read problem statements carefully.**

Problem	Grade	Max
1		20
2		25
3		25
4		15
5		15
Total		100

(1) Let

$$\mathbf{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \text{ and } \mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix} \right\}.$$

Find the relative coordinate vector $[\mathbf{v}]_{\mathcal{B}}$.

(2) Let

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix}.$$

(a) Compute $\det A$.

(b) Compute $\det(A^2)$.

(3) (a) Let \mathbb{P}_3 be the vector space of polynomials (in “ t ”) that are of degree at most 3. Find a basis for \mathbb{P}_3 .

(b) Let $T: \mathbb{P}_3 \rightarrow \mathbb{R}$ be the linear transformation defined by

$$T(\mathbf{p}(t)) = \mathbf{p}(2).$$

Find a basis for the kernel of T .

- (4) Let \mathbf{D} be the parallelogram in \mathbb{R}^2 whose vertices (in clockwise order) are $(1, 3)$, $(3, 6)$, $(7, 5)$, and $(5, 2)$. Find the area of the region enclosed by \mathbf{D} .

- (5) Let A and B be 3×3 matrices that are of rank 1. Prove that the rank of the matrix $A + B$ cannot be greater than 2.