Math 312
Fall 2009
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Proof Assignment #1

Write up solutions to **two** of the following three problems. The due date for this assignment is **Thursday, October 22nd**. This assignment can be done individually or in groups of up to three people.

Problems:

1. Let a be an integer, and let $b = a^3 + 3a^2 + 2a$. Prove that b must be divisible by 6.

2. Let m and n be coprime integers, both greater than or equal to 2. Consider the set of all rational numbers x that can expressed in the form

$$x = \frac{r}{m} + \frac{s}{n},$$

with $r, s \in \mathbb{Z}$. How many such numbers are there that are greater than 0 and less than 1? Prove your answer.

3. Let d be an odd positive integer. Prove that

$$\binom{d}{0} + \binom{d}{2} + \binom{d}{4} + \cdots + \binom{d}{d-1} = 2^{d-1}.$$

Guidelines:

- When writing your proofs, you can assume any results that are proved in Chapters 1-5 in the textbook.[1] You can also assume any results that were proved in class.
- Make your solutions self-contained. The reader should be able to follow your proof without having to look back at the assignment sheet. (A simple way to make your solutions self-contained is to copy the problem down at the beginning of your solution.)
- If you consult any references other than the textbook, indicate that you have done so. (Example: "Sources consulted: *Algebra* by Serge Lang.") If you get help on the assignment from anyone outside of your working-group, you should note that also.

Tips:

- Use complete sentences.
- After writing out a proof, read it to yourself from beginning to end. Note any portions of the proof that are hard to read or not fully justified.
- Don't create your proofs by patching together sentences from the textbook. Proofs that are written this way are hard to read. (Also, copying from a source without proper credit is unethical.) Construct your own sentences.
- Feel free to come to office hours to discuss this assignment. I'm happy to look at a draft of a proof and give you suggestions.

[1]However, please do not assume results that are only stated in the exercises in the textbook.