

Problem

Let x and y be real numbers. Let

$$\dots, F_{-2}, F_{-1}, F_0, F_1, F_2, \dots$$

be the doubly-infinite sequence that satisfies $F_0 = x$, $F_1 = y$, and $F_k = F_{k-1} + F_{k-2}$ for all $k \in \mathbb{Z}$. Show that

$$\mathbf{inf} \{|F_k| : k \in \mathbb{Z}\} \leq \sqrt{\frac{|x^2 + xy - y^2|}{5}}.$$

(Math Problem of the Week, June 15, 1997)

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