

Problem

Let L be the set of lattice points on a 2-dimensional Cartesian coordinate system. We shall call a planar line a “lattice line” if it passes through an infinite number of lattice points.

- (a) Determine whether there exists a subset $S \subseteq L$ such that every lattice line passes through exactly 2 members of S .
- (b) Show that there exists a function $f: L \rightarrow [0, 1]$ such that for any lattice line Q , the sum of $f(a, b)$ over all points (a, b) on Q is 1.
- (c) Determine whether there exists a function satisfying the above conditions with $f(a, b) > 0$ for all (a, b) .

*(Math Problem of the Week, 7/21/96)
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