

Problem

For any positive integer n , let \mathbf{P}_n denote the set of sequences of positive integers which are nonincreasing and which sum to n . For any sequence $S \in \mathbf{P}_n$, let $f(S)$ be the product of the terms of S multiplied by the factorials of the number of times each distinct term appears. For example,

$$f(5, 3, 3, 2, 2, 2, 1) = (5 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot 1)(1! \cdot 2! \cdot 3! \cdot 1!) = 4320.$$

Prove that for any n ,

$$\sum_{S \in \mathbf{P}_n} \frac{1}{f(S)} = 1.$$

(Math Problem of the Week, 8/11/96)
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