

Problem

Let $f: \mathbb{Z} \rightarrow \mathbb{Z}$ be a function satisfying the following two conditions:

- (1) $f(n+a+b) - f(n+a) - f(n+b) + f(n) \in \{-1, 0, 1\}$ for all n , a , and b .
- (2) For all n and a , there are an infinite number of values for b such that $f(n+a+b) - f(n+a) - f(n+b) + f(n) = 0$.

Show that for some pair of reals (x, y) , either $f(n) = \lfloor xn + y \rfloor$ for all n , or $f(n) = \lceil xn + y \rceil$ for all n .

(Math Problem of the Week, 8/18/96)

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