

Problem

Let x and y be real values in the interval $[0, 1)$, and let a and b be relatively prime positive integers. Show that there exist real values z and w such that

$$az - \lfloor az \rfloor = x,$$

$$bw - \lfloor bw \rfloor = y,$$

and

$$|z - w| \leq \frac{1}{2ab}.$$

(Math Problem of the Week, 9/22/96)

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