Problem

Let S_0, S_1, S_2, \ldots be a sequence of positive integers such that $(S_{i-1} + S_{i+1})/S_i$ is an integer for every $i \geq 1$. Show that one of the following conditions must hold:

- (i) (S_i) contains a term equal to $gcd(S_0, S_1)$.
- (ii) $\lim_{i\to\infty} S_i = \infty$.

("Power question" for the Duke Math Meet, 1998) Carl Miller