Math 217
Section 003
Winter 2008
Carl Miller

Proof assignment #1

Write up solutions to **three** of the four problems below. The due date for this assignment is **Friday, March 14th**.

Problems:

1. Let A be an $n \times n$ matrix, and let S be the set of vectors $\mathbf{v} \in \mathbb{R}^n$ such that $A\mathbf{v} = 2\mathbf{v}$. Show that S is a subspace of $\mathbb{R}^n$.

2. Suppose that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a basis for $\mathbb{R}^3$. Show that $\{\mathbf{v}_1, \mathbf{v}_2 + \mathbf{v}_1, \mathbf{v}_3 + \mathbf{v}_1\}$ is a basis for $\mathbb{R}^3$.

3. Suppose that B is an $m \times m$ matrix such that the sum of the entries on any row of B is zero. Prove that the linear transformation $\mathbf{x} \mapsto B\mathbf{x}$ cannot be onto.

4. Suppose that C is a 3×3 matrix such that for any other 3×3 matrix D, CD is equal to DC. Show that $C = tI_3$ for some real number t.

Guidelines:

- When writing your proofs, you can assume that the reader is familiar with chapters 1 and 2 in our textbook. (You can also assume that the reader has all of the prerequisite knowledge for this course.) In your proofs you can use any results that are stated in the text of chapters 1 and 2.[1]
- If you consult any references other than the textbook, indicate that you have done so. (Example: "Sources consulted: *Algebra* by Serge Lang.") If you get help on the assignment from anyone besides me (if, for example, you talk about the problems with another Math 217 student) you should note that also.
- Make your solutions self-contained. The reader should be able to follow your proof without having to look back at the assignment sheet. (A simple way to make your solutions self-contained is to copy the problem down at the beginning of your solution.)

Tips:

- Use complete sentences.
- After writing out a proof, read it to yourself from beginning to end. Note any portions of the proof that are hard to read or not fully justified.
- Don't create your proofs by patching together sentences from the textbook. Proofs that are written this way are hard to read. (Also, copying from a source without proper credit is unethical.) Construct your own sentences.
- Feel free to come to office hours to discuss this assignment. I'm happy to look at a draft of a proof and give you suggestions.

[1] However, please do not use results that are only stated in the **exercises** in chapters 1 and 2.