Math 567
Winter 2009
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Problem Set #2

Due date: Wednesday, February 11th.

1. Find a self-dual binary $[6,3]$-code.

2. Suppose that n and d are positive integers with $d > 2n/3$. Prove that does not exist a binary $[n,2,d]$-code.

3. Suppose that a message consisting of 6 zeroes is sent through a binary symmetric channel with bit-error probability $p = 0.1$. What's the probability the received message contains exactly three 0's and three 1's?

4. **(a)** Let $C \subseteq \mathbb{F}_2^7$ be the code with generator matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Find a parity check matrix for C.

(b) Suppose that this code is used on a binary symmetric channel. Suppose that a 7-bit codeword $\mathbf{x} \in C$ is sent across the channel, and the received word $\mathbf{x}'$ is the same as $\mathbf{x}$ except that the 5th bit is flipped. What's the syndrome of $\mathbf{x}'$? (Use the parity check matrix you found in part (a).)

5. Prove that there is no such thing as a perfect binary $[16,k,3]$-code.

6. *(Extra credit.)* How many binary $[n,2]$-codes exist? (In other words, count the number of subspaces of dimension 2 in $\mathbb{F}_2^n$. Express your answer as a formula involving n.)