

Problem Set #3

**Due date:** Wednesday, March 4th.

1. Let  $C \subseteq (\mathbb{F}_2)^5$  be the linear code with generator matrix

$$M = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

Find the weight enumerator for  $C$ .

2. Let  $K = \mathbb{F}_3[X]/(X^3 + 2X + 1)$ , and let  $\alpha$  be the element represented by  $X$  in  $K$ . Compute the following. Express your answers as linear combinations of  $1$ ,  $\alpha$ , and  $\alpha^2$ .

- (a)  $\alpha^4$
- (b)  $\alpha^6$
- (c)  $\alpha^{-1}$

3. Let  $C \subseteq (\mathbb{F}_5)^4$  be the one-dimensional subspace generated by the vector  $(1, 2, 2, 3)$ . Find a generator matrix in standard form for  $C^\perp$ .

4. Let  $G$  be the group of bijective maps

$$f: \{1, 2, 3, 4, 5\} \rightarrow \{1, 2, 3, 4, 5\}$$

such that  $f(1) = 1$ . (Multiplication in this group is given by composition.)

- (a) Is this an abelian group?
- (b) What is  $|G|$ ?
- (c) What is the identity in this group?
- (d) Let  $e \in G$  denote the identity. Find an example of an element  $f \in G$  such that  $f \neq e$  but  $f^2 = e$ .

5. Let  $F$  be a finite field, and let

$$S = \{a^3 \mid a \in F\}$$

( $S$  is the set of cubes in  $F$ .) How large is  $S$  if  $F = \mathbb{F}_{131}$ ? If  $F = \mathbb{F}_{109}$ ? If  $F = \mathbb{F}_{125}$ ? Explain your reasoning.

6. (*Extra credit*) Find the weight enumerator for a binary Hamming code of length  $2^k - 1$ .