Math 567
Winter 2009
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Problem Set #4

Due date: Friday, March 20th.

1. Let $\mathcal{H}$ be the linear code over $\mathbb{F}_2$ with generator matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

$\mathcal{H}$ is a binary Hamming code. Its minimum distance is 3.

(a) Suppose that we puncture $\mathcal{H}$ by deleting the last position. What are the parameters (including minimum distance) of the new code?

(b) Suppose that we extend $\mathcal{H}$ by adding a parity check bit. What are the parameters of the new code?

(c) Suppose that we shorten $\mathcal{H}$ by one position. What is the mininum distance of the new code?

2. Let C_1 and C_2 be linear codes over $\mathbb{F}_2$. Let C be the *direct product* of C_1 and C_2 (see Problem 3.8.12 in the textbook for the definition of direct product). Is it possible to determine the weight distribution of C from the weight distribution of C_1 and C_2? Explain.

3. Let $\mathcal{G}_{23}$ denote the binary Golay code. What is the minimum distance of $\mathcal{G}_{23}^{\perp}$? (Explain your answer.)

4. Let $q = p^r$, where p is prime.

(a) Count the number of invertible $d \times d$ matrices with entries from $\mathbb{F}_q$.

(b) *(Extra credit)* Count the number of d-dimensional subspaces of $(\mathbb{F}_q)^n$.

5. Let n be an even positive integer. Is it is possible to find $n+1$ codewords in $\mathbb{F}_2^n$ such that any two of them differ in exactly $n/2$ places? Find a general approach, or prove that this is not possible.

6. (a) How many codewords of weight 6 are contained in the ternary Golay code?

(b) *(Extra credit)* Compute the full weight enumerator of the ternary Golay code.