Problem Set #5

Due date: Wednesday, April 1st.

1. What is the weight enumerator for the Reed-Muller code $\mathcal{R}(1, m)$?

2. Suppose that a binary code with minimum distance 5 is punctured twice. What are all possible values for the new minimum distance?

3. Prove that there cannot exist a linear $[11, 4, 6]$-code over $\mathbb{F}_2$.

4. Let $a(X) = X^4 + X^3 + X + 1 \in \mathbb{F}_2[X]$.

(a) Find a factorization of $a(X)$ into irreducible polynomials.

(b) How many ideals does the ring $\mathbb{F}_2[X]/(a(X))$ contain?

5. Prove that the Reed-Muller code $\mathcal{R}(m - 1, m) \subseteq (\mathbb{F}_2)^{2^m}$ is the set consisting of all even-weight vectors.

6. (a) What is the value of $A(7, 3)$ for $q = 2$? Justify your answer.

(b) *(Extra credit)* What is the value of $A(6, 3)$ for $q = 2$? Justify your answer.