Problem

Let T be a finite set. Suppose that S is a set of pairs of subsets of T which satisfies the following condition:

- For any $T' \subseteq T$, there exists exactly one pair $(X, Y) \in S$ such that $X \subseteq T' \subseteq Y$.

Assume that $(\{\}, T) \notin S$. Show that there exist two distinct pairs (A, B) and (A', B') in S such that $B \smallsetminus A = B' \smallsetminus A'$.

(Math Problem of the Week, 9/29/96)
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